Workshops:
The heart of the MagiKats Programme

Every student is assigned to a Stage, based on their academic year and assessed study level.

Stage 4 students are approximately 11 to 14 years old.

The sheets in this pack are a small sample of what is available! These are only samples of the student's worksheets - our teaching methods include discussion and hands-on activities.

Core skills sheets are also provided for independent completion by each student (usually at home).

At this level, students study all the topics needed to take them to about age 16. Further material in Stage 5, offered from age 14, is completed by those expecting to continue studying mathematics to a more advanced level.

MATHS STAGE 4

MagiKats
TUITION CENTRES

Data Handling: The Basics

Modern life continually demands decisions of us: what job do we want to have, what subjects shall we study at school, how shall we pass our spare time, how do we spend our money? Many of these questions can be answered quite simply, but many problems cannot be decided without further information. This information can be displayed in many different ways, some of which are more help to us than others.

Let's do a survey.
We looked at the registration letter of the cars in a school car park. They were:

Complete the table below to show the frequency of each registration letter.

Registration letter	Tally	Frequency
J		
K		
L	/	
M		
N	//	
P		
R	/	
S	/	
		Total

Your total should be 17. If it is not then go back and check your answer.

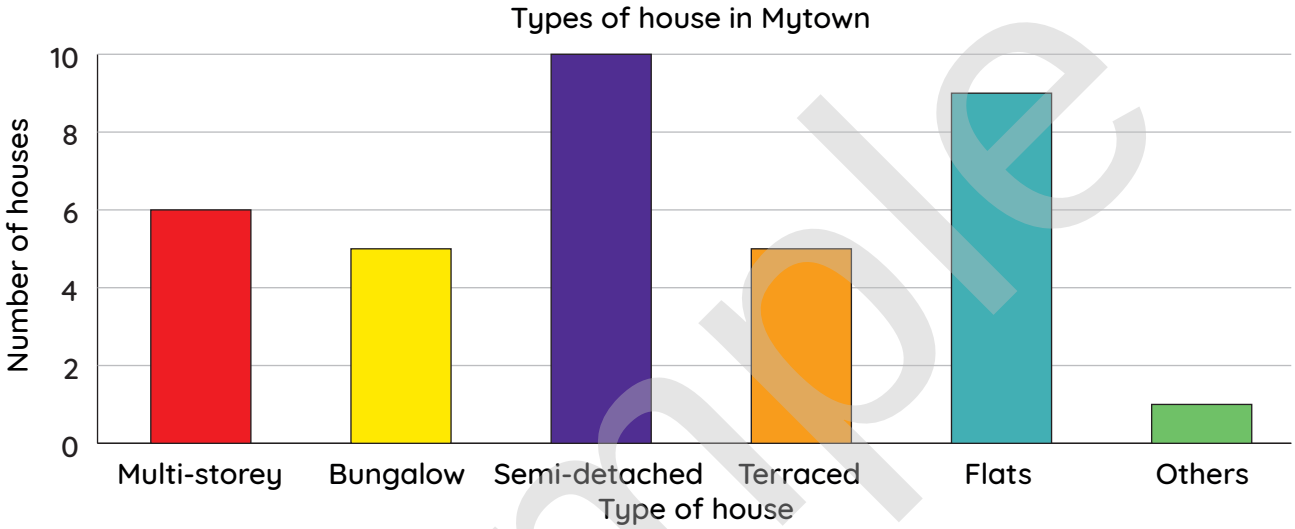
The frequency that a particular item of data appears is very important. When faced with lots of data, this is often the easiest way of putting it together. This is known as a **Tally Chart**.

In a tally, every 5th mark crosses a group of 4 like this:
This is known as a five-bar-gate.

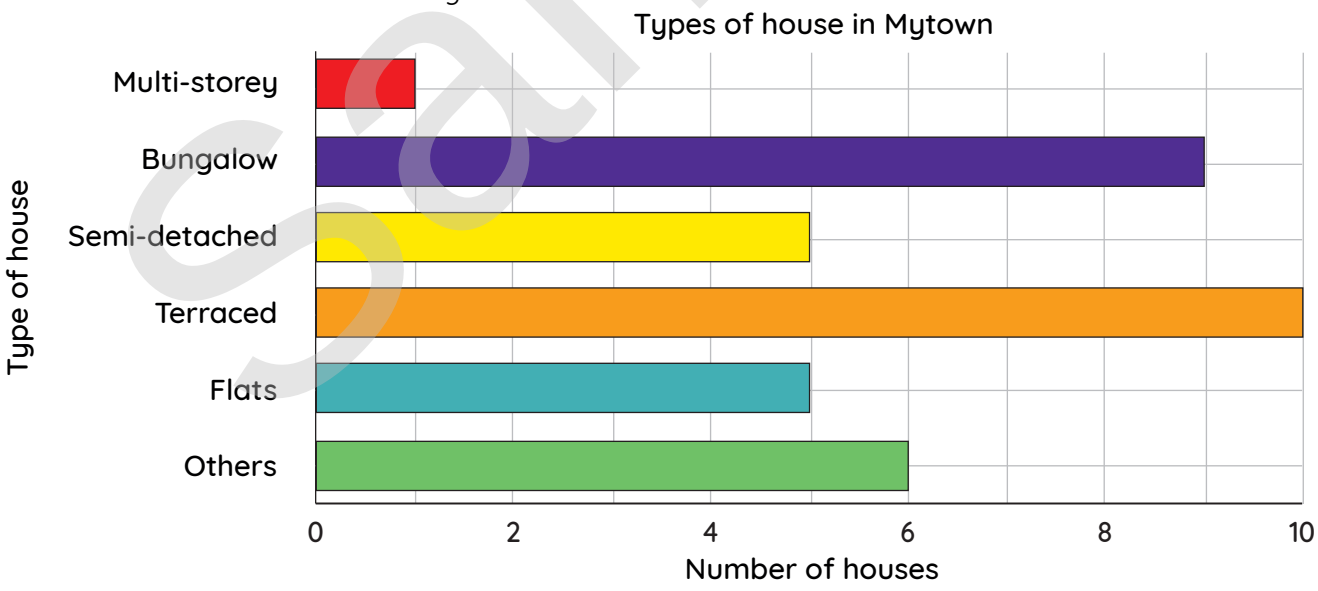
Bar Charts

Another very common way of presenting this information is in a **Bar Chart**.
A bar chart is a way of presenting data using a series of rectangular bars.
What you must watch for is when the bars touch and when they do not.

If you are comparing totally separate items, the bars must have gaps between them. If the items are related (for example, increasing lengths), the bars must not have gaps between them.



This chart can also be drawn sideways.



There are certain things to remember when drawing a bar chart.

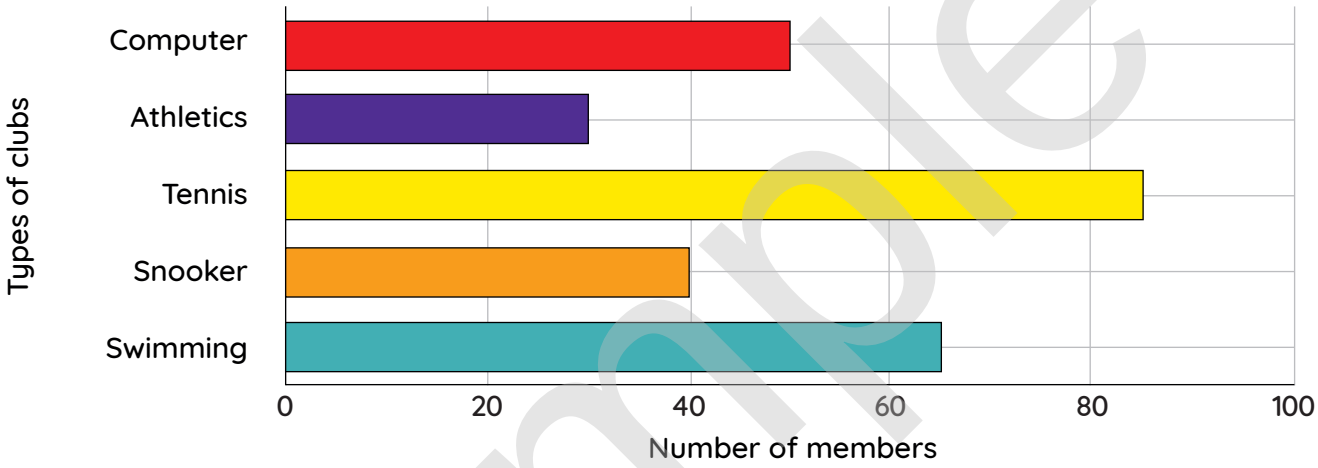
1) **The chart should always have a heading, describing exactly what information is illustrated.**
2) **Each axis should be labelled with the scale and description of the data.**

Now you have learnt the basics of bar charts, complete the questions on the next sheet for some practice.

1) On some graph paper, draw a bar graph showing how David's farm is divided.

Crop	Grass	Barley	Oats	Linseed	Rapeseed
Acres	90	30	30	10	20

2) This bar chart shows the number of members for five different clubs at school.



a) How many people belong to the athletics club? _____

b) Which is the most popular club? _____

c) How many more people belong to the swimming club than the computer club?

_____

3) This table gives information, from food wrappers, of fat and protein content in some popular products.

a) Draw a bar chart to show this information. Remember to give a key showing which bars represent fat and which ones protein.

Product	Soup	Oven Chips	Breakfast Cereal	Yoghurt	Mayonnaise
Fat per 100g	3g	5g	6g	1g	10g
Protein per 100g	1g	2g	9g	5g	1g

b) Which product has most protein per 100g? _____

c) Which product has $2\,{}^{1}/_{2}$ times more fat than protein? _____

Pie Charts

Another way of displaying the results of a survey is by using a Pie Chart.
This is where a circle is divided up into slices (sectors) of different sizes.

In the survey already discussed, there were 36 people questioned altogether.
Can you remember how many degrees there are in a circle?

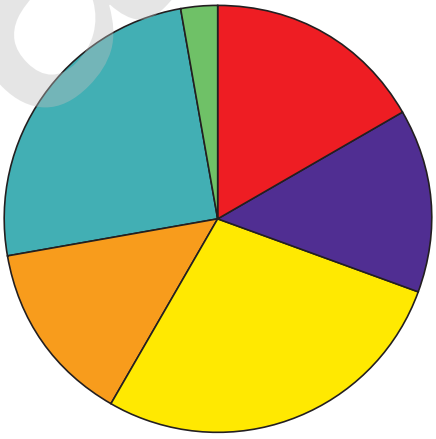
That's right: 360°, which means each person has a 360° ÷ 36 = 10° sector of the whole pie.
The 6 people who live in multi-storey detached houses will therefore altogether need a sector measuring 60°.

Work out the angular measurements for the groups.

	Multi-storey	Bungalow	Semi-detached	Terraced	Flats	Others	**Total**
Number of people	6	5	10	5	9	1	**36**
Number of degrees in pie chart							

We now use a protractor to measure these angles at the centre of a circle. A suitable radius is usually 5 or 6 cm. This pie chart has been drawn for you. All you need to do is label the sectors.

Types of house in Mytown



Measure the angles at the centre for each sector and check that this chart agrees with the table above.

A pie chart makes it really easy to distinguish the big sectors from the small ones and it also is really easy to judge by eye what fraction each one represents. This is often the most important aspect of a survey.

Now complete the following questions for some practice.

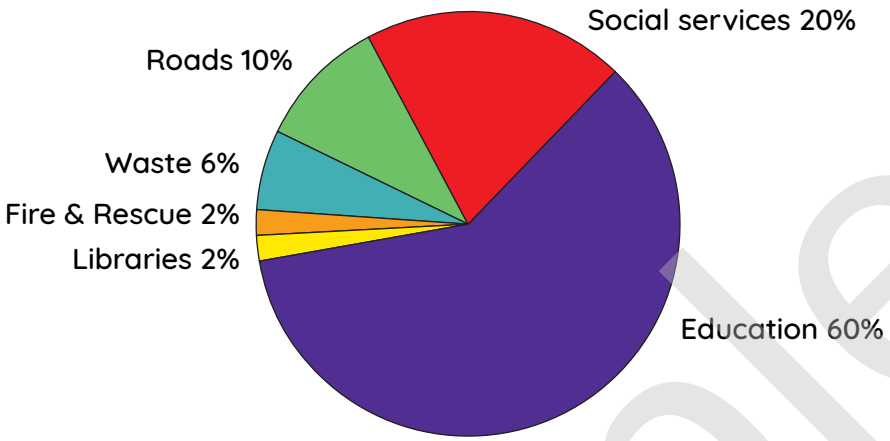
1) Complete this table and then draw a pie chart to illustrate the information.

Favourite TV Soap	Number of Viewers (millions)	Degrees in sector
Riverside	15	30°
Westenders	45	
Far and Away	30	
Grimdale	30	
Abdication Street	60	
Totals		

2) Draw a pie chart to show the following information about David's farm.

Animal	Cattle	Sheep	Horse	Chicken	Ostrich
Number	**60**	180	40	70	10

3) This diagram shows how a local council spent its budget.

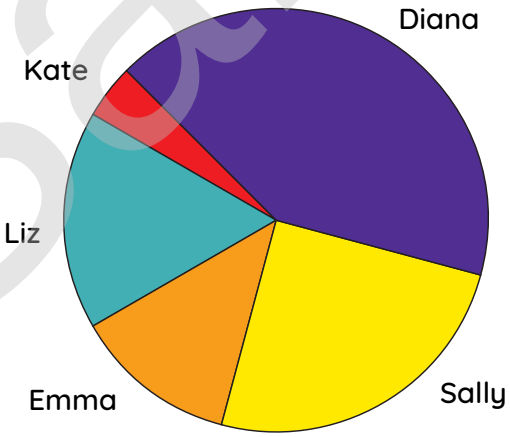


a) What fraction of the budget was spent on education? _____

b) What fraction of the budget was spent on Fire & Rescue? _____

c) Calculate how many degrees represent roads. _____

4) This pie chart shows the proportions of votes for candidates in the local election.



a) Measure and write down the angles in the pie chart.

_____

b) What fraction of people voted for Sally? _____

c) If 180 people voted for Sally, how many voted in total? _____

d) How many voted for Diana? _____

A Summary

Now you have learned about some of the main ways of handing and representing data, let's fill in this summary table, which you can take home with you.

Type of chart	Features	When to use
Tally chart		
Pictogram		
Bar chart		
Pie chart		

Factors

A term that is used regularly in maths is **factor**. What does this mean?

List out as many factors as you can think of for each number below.

12

20

63

13

144

32

31

Now for a harder one! Show all your working and think carefully about the best way to approach the task. Be ready to explain this to your mentor.

Find all the factors of 49 600.

To break a number down in to its factors, you need to be systematic.

Look at **49 600**

Start to break it down to make it more manageable **49 600 = 496 x 100**

Concentrate on 496. It is even so **496 = 2 x 248**

Keep looking! **248 = 8 x 31**

31 has no factors except itself and 1.

8 = 2 x 2 x 2

So **496 = 2 x 2 x 2 x 2 x 31**

100 = 2 x 2 x 25 = 2 x 2 x 5 x 5

Therefore,

49 600 = 2 x 2 x 2 x 2 x 2 x 2 x 5 x 5 x 31

There are several ways that you could approach this but, in every case, you are systematically breaking the number down. It helps if your tables are good. You will be working on another approach later in this set, on the way to finding Greatest Common Factors.

We said that 31 has no factors except itself and 1.

What do we call this sort of number? _____

Prime Numbers

A prime number is defined as a number that is greater thatn 1 and has exactly two factors, itself and 1.

Find the factors of each number below, then list those that are prime.
Circle any number that is prime.

1) **26**

Factors: _____

Prime Factors: _____

2) **9**

Factors: _____

Prime Factors: _____

3) **114**

Factors: _____

Prime Factors: _____

4) **110**

Factors: _____

Prime Factors: _____

5) **41**

Factors: _____

Prime Factors: _____

6) **105**

Factors: _____

Prime Factors: _____

Greatest Common Factor (GCF)

Now let's look at pairs of numbers, say 18 and 30.

Factors of **18** are **1, 18, 2, 9, 3, 6**

Factors of **30** are **1, 30, 2, 15, 3, 10, 5, 6**

Common factors are **1, 2, 3, 6**

The greatest common factor is **6**

When working with any pair of numbers, it is often useful to be able to identify their **Greatest Common Factor (GCF)**. This is the largest number that will divide into both numbers without a remainder.

You may be able to 'spot' the GCF or work it out as above but, if not, this method is quick and simple:-

Write down the two numbers, separated by a comma, as a division sum.

Identify a number that will divide into both and complete division.

Repeat.

Continue until you can go no further.

2)18 , 30

3) 9 , 15

3 , 5

The GCF is the product of the numbers that divided exactly.

So, the GCF of 18 and 30 is 2 x 3 = 6

Knowing this would allow you to reduce fractions in one step.

On a separate piece of paper, use this method to find the GCF of the following numbers.

1) (150, 210)

2) (18, 63)

3) (180, 216)

4) (24, 60)

5) (24, 32)

6) (9, 21)

7) (64, 224)

8) (6426, 1071)

9) (960, 1440)

Lowest Common Multiple (LCM)

A partner value to the Greatest Common Factor is the **Lowest Common Multiple (LCM)** which is used in fractions as the lowest common denominator. This is the smallest number that both numbers can be multiplied up to using whole numbers.

You can use the same technique – see below.

2) 16 , 20

2) 8 , 10

4 , 5

Here the GCF is 2 x 2 = 4

To find LCM, multiply this by the values remaining after all possible divisions (here 4 and 5).

LCM of 16 and 20 is 2 x 2 x 4 x 5 = 80

Example: Find the GCF and LCM of 48 and 72

2) 48 , 72

2) 24 , 36

3) 12 , 18

2) 4 , 6

2 , 3

GCF = 2 x 2 x 2 x 3 = 24

LCM = 2 x 2 x 2 x 3 x 2 x 3 = 144

On a separate piece of paper, use this method to find the LCM of these pairs of numbers.
You might find it useful to refer back to your workings for sheet 5.

1) (150, 210)

2) (18, 63)

3) (180, 216)

4) (24, 60)

5) (24, 32)

6) (9, 21)

7) (64, 224)

8) (6426, 1071)

9) (960, 1440)

Powers

We worked out that 49 600 = 2 x 2 x 2 x 2 x 2 x 2 x 5 x 5 x 31

Instead of writing it out like this we could have used mathematical shorthand.

We could have written 49 6000 = 25^5 x 52^2 x 31

2^5 means 2 x 2 x 2 x 2 x 2 this is described as 2 to the power 5
5 is described as the power or the index number.

Calculate, without a calculator

1) 2^6 2) 2^7 3) 2^8

4) 5^3 5) 7^4 6) 10^5

If you have not already done so, now would be a good time to work out the short cut (function) key on your calculator! Use it to check your answers to the last three questions and to work out the answers to the next three. Give answers to 3 decimal places.

7) 3.12^3 8) 7.34^5 9) 9.813^{12}

Now use your brain! What would be the value, expressed with a single power, of the following:

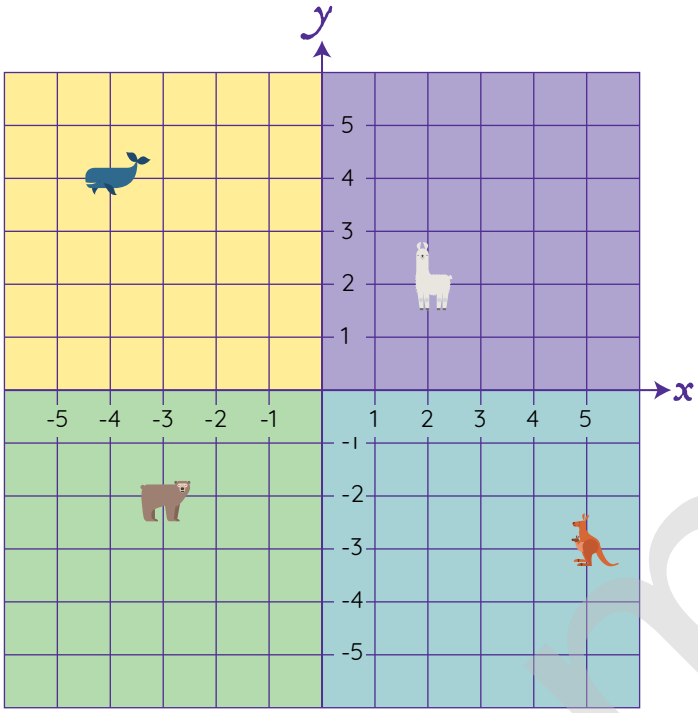
10) 6^4 x 6^5 x 6^4 =

11) 12.5 x 12.5^6 x 12.5 x 12.5^7 =

12) 0.7^4 x 0.7^3 x 0.49 =

Linear Functions

A graph has four different regions where the x and y coordinates are either positive or negative. Look carefully at the graph below and work out whether each of the x and y coordinates is positive or negative in each region.



Complete this table

Region	Is x + or -?	Is y + or -?
Llama		
Whale		
Bear		
Kangaroo		

Find the creatures...
Write down the coordinates of each animal. Remember: x coordinate then y coordinate.

Creature	x coordinate	y coordinate	written
Llama			(,)
Whale			(,)
Bear			(,)
Kangaroo			(,)

Label the axes on your sheet of graphs then:

On graph 1: join, in red, all the points where x = 3 and, in blue, all the points where x = -2.

On graph 2: join, in red, all the points where y = 3 and, in blue, all the points where y = -2.

On graph 3: join, in red, all the points where y = x and, in blue, all the points where y = -x.

On graph 4: join, in red, all the points where y = 3x and, in blue, all the points where y = $^1/_2$x.

Remember to label all your lines clearly.
Use a ruler and keep it tidy.

Drawing straight line graphs from equations means substituting values in to equations — like in algebra!

Example: Draw the graph of $y = 3x - 1$ for $0 \leq x \leq 4$.

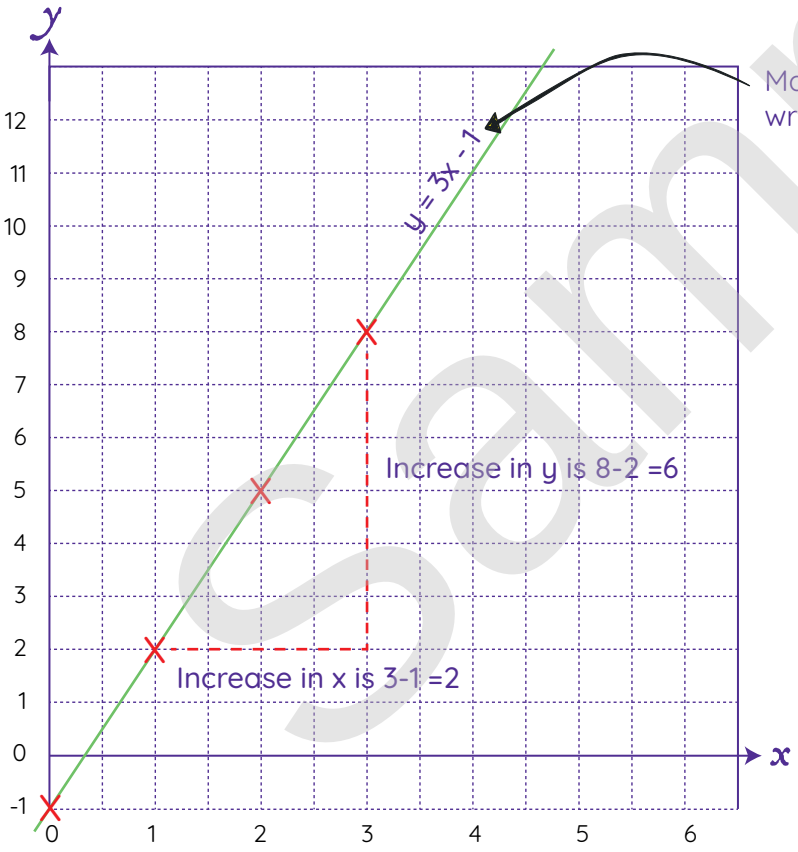
Step 1 Draw and complete a table of values:

x	0	1	2	3	4
y	-1	2	5	8	11

← specified in question

← calculated from $y = 3x - 1$

Step 2 Draw the axes (use graph paper)
Step 3 Plot the points
Step 4 Join the points (use a ruler as it is a straight line) and label the graph.



Make sure your equation is written by or along the line.

Points to note:
At the point where the graph cuts the y axis, x = 0 and y = -1

Gradient = $\frac{\text{increase in y}}{\text{increase in x}} = \frac{6}{2} = 3$

Now look at the equation of the graph $y = 3x - 1$

Where will $y = 2x + 1$ cut the y axis and what will its gradient be?
Plot it to check your answer!

From the previous page, you can see that there must be a formula to work out the equation of a line.

Here it is:

$$y=mx+c$$

What does it mean?

m This is the gradient of the line.

c This is the point at which the graph cuts the y axis (also called the y-intercept)

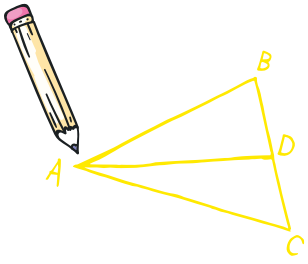
Let's take this equation:

$$y=4x+2$$

What's the gradient (the 'm' value)? _____

What's the y-intercept (the 'c' value)? _____

Try the questions on the next page.



$AB^2 = AC^2 + CB^2$

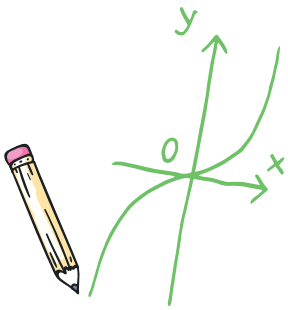
Complete this table:

Equation	Gradient of graph	Intercept on y axis
y = 4x + 2		
y = 5x - 1		
y = 6x		
y = 2x + 5		
y = 12 – 3x		
y = x		
y = 3 – x		
y = 10 – 2x		
y = 4x - 5		
2y = x + 4		

Now draw each of the graphs to check your answers.

Use the method on Sheet 3 to complete a table of values, plot the points and draw the graph. On each graph, mark the intercept then sketch in a triangle to work out the gradient.

You might think that, for a straight line, you only need to plot two points but you really need to plot at least three to guard against mistakes.



$y = x^2$

Graphs are often used to solve problems. When you are drawing graphs, you need to take care with your axes and your choice of scales.

Read this information:

A greyhound runs 400m in 25 seconds. Draw a graph, on graph paper, to show this information. Plot time on the x axis and distance from the start on the y axis. Use the available space as far as possible and take care with your scales.

Draw in a triangle to calculate the gradient of the line (take care with units!).

The gradient =

In this example the gradient represents distance travelled in a certain time.

What would we usually call this? _____

Write down the equation of the line. _____

The mechanical hare, that the greyhound chases, is given 100m start and the dog catches the hare after it (the dog) has travelled exactly 400m.

Draw a graph to show this.

How fast does the hare run? _____

Number Patterns and Sequences

Look at each of the sequences below and see if you can work out the next two values.
Make a clear note of how you worked them out.

2, 5, 8, 11, 14

30, 24, 18, 12

8, 11, 15, 20, 26

53, 43, 34, 26, 19

5, 10, 20, 40

2, 6, 18, 54

400, 200, 100, 50

40000, 4000, 400, 40

1, 1, 2, 3, 5, 8, 13

When you have answers to all of these, look at the next page and see if you were right!

These are the sequences and rules that you should have found!

Adding or subtracting the same number

Write the differences in the gaps between each pair of numbers:

2 → 5 → 8 → 11 → 14
+3 +3 +3 +3

Next two values 17, 20
Rule: Start from 2.
Add 3 to the previous term.

30 → 24 → 18 → 12
-6 -6 -6

6, 0
Start from 30.
Take 6 from the previous term.

Adding or subtracting a changing number

Write the change in the gaps:

8 → 11 → 15 → 20 → 26
+3 +4 +5 +6

Next two values 33, 41
Rule: Start from 8 by adding 3
Add 1 extra each time to previous term

53 → 43 → 34 → 26 → 19
-10 -9 -8 -7

13, 8
Start from 53 by subtracting 10
Subtract 1 less from previous term

Multiplying by the same number each time

Write the common multiplier between each pair of numbers:

5 → 10 → 20 → 40
x2 x2 x2

Next two values 80, 160
Rule: Start from 5
Multiply the previous term by 2

2 → 6 → 18 → 54
x3 x3 x3

162, 486
Start from 2
Multiply the previous term by 3

Dividing by the same number each time

Write the common divider between each pair of numbers:

400 → 200 → 100 → 50
÷2 ÷2 ÷2

Next two values 25, 12.5
Rule: Start from 400
Divide the previous term by 2

40000 → 4000 → 400 → 40
÷10 ÷10 ÷10

4, 0.4
Start from 40000
Divide the previous term by 10

Adding the previous two terms

This type of sequence works by adding the last two numbers to get the next one.

e.g. 1 → 1 → 2 → 3 → 5 → 8 → 13
1+1 1+2 2+3 3+5 5+8

Next two values 21, 34

Rule: Add the previous two terms

How did you get on?

The sequence 3, 9, 15, 21... starts at 3 and has a common difference of +6.

Concentrate on one term and see how it is worked out, e.g. the fourth term which is 21.

Value of first term = 3
Common difference = +6
Number of term = 4 (fourth term)
Value of term = 21

The calculation to reach 21 was $3 + \underline{6 + 6 + 6}$

First term + 3 x common difference, i.e. (4 – 1) x 6

Using this approach for the 99th term we have 3 + 98 x 6 = 591

From this we can see n^{th} term = first term + (number of term minus 1) x common difference.
so

$$n^{th} \text{ term} = a + (n-1) \times d$$

first term common difference

To find the n^{th} term, find the values of 'a' and 'd' and put them into the formula.
You don't replace n though, that stays as n.

Example:

Find the n^{th} number of this sequence: 5 8 11 14

Answer:

1) The formula is $\underline{a + (n-1) \times d}$

2) The first number is 5, so $\underline{a = 5}$. Common difference is 3 so $\underline{d = 3}$.

3) Putting these in the formula gives: $5 + (n-1) \times 3$
$= 5 + 3n - 3$
$\underline{= 3n + 2}$

$\underline{n^{th} \text{ term} = 3n + 2}$

To find the value of a particular term, we do put in a value for n.
For the 8^{th} term, n = 8 so 8^{th} term $= 3 \times 8 + 2$
$= 26$

Suddenly all that algebra and substitution practice looks useful!

Using the formula:

1) Find the 12^{th} term of the series 5, 8, 11, _____

2) Find the 13^{th} term of the series 80, 75, 70, _____

3) Find the 50^{th} term of the series 100, 97, 94, _____

4) Find the 23^{rd} term of the series -7, -3, +1, _____

5) Find the 10^{th} term of the series 18, 14, 10, _____

6) Find the n^{th} term of the series 10, 14, 18, _____ (This is a formula, not a number)

7) Find the n^{th} term of the series 18, 12, 6, _____

8) Find the n^{th} term of the series 6, 13, 20, _____

1) In each of the questions below, write the next three numbers in the sequence and then the rule you used.

a) 1, 3, 5, 7, _____ , _____ , _____ Rule _____

b) 2, 4, 8, 16, _____ , _____ , _____ Rule _____

c) 3, 30, 300, 3000, _____ , _____ , _____ Rule _____

d) 3, 7, 11, 15, _____ , _____ , _____ Rule _____

e) 6, 15, 24, 33, _____ , _____ , _____ Rule _____

f) 19, 14, 9, 4, -1, _____ , _____ , _____ Rule _____

2) These are well known number sequences. Write the next two numbers and name the sequence.

a) 1, 4, 9, 16, 25, 36, _____ , _____ Name _____

b) 1, 3, 6, 10, 15, 21, _____ , _____ Name _____

3) The letter n describes the position of a term in the sequence. If n = 1, that's the 1st term – if **n** = 10 that's the 10th term and so on. In the following questions, use the given rule to generate the first 5 terms.

a) 3**n** + 1 so if **n** = 1 the 1st term is (3 x 1) + 1 = 4

n=2 the 2nd term is _____ = _____

n=3 _____ = _____

n=4 _____ = _____

n=5 _____ = _____

b) 5**n** –2, when **n** = 1, 2, 3, 4 and 5 produces the sequence _____ , _____ , _____ , _____ , _____

c) $\mathbf{n}^2$, when **n** = 1, 2, 3, 4 and 5 produces the sequence _____ , _____ , _____ , _____ , _____

d) $\mathbf{n}^2$-3, when **n** = 1, 2, 3, 4 and 5 produces the sequence _____ , _____ , _____ , _____ , _____

e) (**n** +2)÷2, when **n** =1, 2, 3, 4 and 5 produces the sequence _____ , _____ , _____ , _____ , _____

4)

a) Find the next two terms in this sequence: 3 (+4) 7 (+4) 11 (+4) 15 _____ _____

b) The difference between the terms is always _____ .

To find the n^{th} number in the sequence you can use the rule **4n** – 1.
4n is **n** x 4 the difference between the terms.
-1 is found by subtracting the difference between the terms from the first term 3 –4.

What will the 10^{th} number in the sequence be? _____

5) Find the next two terms in this sequence: 8 13 18 23 _____ _____

What is the difference between the terms? _____

Subtract the difference between the terms from the first term. _____

Write down the rule for finding the n^{th} term. _____

What is the 20^{th} term in the sequence? _____

6) In each of the following patterns find:

a) The next two terms of the sequence.

b) The difference between the terms.

c) Subtract the difference between the terms from the first term.

d) The rule for the n^{th} term.

e) The 50^{th} term of the sequence.

6	10	14	18	a)	_____ , _____	b)	_____	c)	_____	d) _____ e) _____
5	8	11	14	a)	_____ , _____	b)	_____	c)	_____	d) _____ e) _____
3	11	19	27	a)	_____ , _____	b)	_____	c)	_____	d) _____ e) _____
6	9	12	15	a)	_____ , _____	b)	_____	c)	_____	d) _____ e) _____

How did you get on?

The sequence 3, 6, 12, 24... starts at 3 and has a common ratio of 2.

Concentrate on one term and see how it is worked out, e.g. the fourth term which is 24.

Value of first term = 3
Common ratio = 2
Number of term = 4 (fourth term)
Value of term = 24

The calculation to reach 24 was $3 \times 2 \times 2 \times 2$

First term x 3 x common ratio, i.e. (common ratio)^3

Using this approach for the 19th term we have $3 \times 2^{(19-1)} = 786432$

From this we can see nth term = first term x common ratio$^{(n-1)}$
so

$$n^{th} \text{ term} = a \times r^{(n-1)}$$

first term common ratio

To find the nth term, find the values of 'a' and 'r' and put them into the formula.
You don't replace n though, that stays as n.

Example:

Find the nth number of this sequence: 5 15 45

Answer:

1) The formula is $a \times r^{(n-1)}$

2) The first number is 5, so $a = 5$. Common ratio is 3 so $r = 3$.

3) Putting these in the formula gives: $5 \times 3^{n-1}$
Remember you must not multiply the 3 by the 5.

$$n^{th} \text{ term} = 5 \times 3^{n-1}$$

To find the value of a particular term, we do put in a value for n.

For the 8th term, n = 8 so 8^{th} term $= 5 \times 3^7$
$= 10935$

As the numbers are often very large, we usually leave answers in index form so 8^{th} term $= 5 \times 37^7$

Try these questions, using the formula. Leave answers in index form.

1) What is the 40^{th} term of the GP 2, 14, 98 _____ ?

2) What is the 10^{th} term of the GP 16, -8, 4 _____ ?

3) What is the 24^{th} term of the GP 5, 15, 45 _____ ?

4) What is the 6^{th} term of the GP 99, 33, 11 _____ ?

5) Find n given that 1024 is the nth term of the GP 4, 8, 16 _____